Granice ciągów – zadania

Zadanie 1

Obliczyć granicę ciągu (a_n), gdy:

(a) $a_n = \dfrac{2n^2-1}{n^2+1}$

(b) $a_n = \dfrac{4n^3-n}{n^2+1}$

(c) $a_n = \dfrac{n^3-n}{3n^5+1}$

(d) $a_n = \dfrac{\binom{n}{2}}{n^2}$

(e) $a_n = \dfrac{2^{n+1}+3^{2n-1}}{4^n+9^n}$

(f) $a_n = \dfrac{2^{-n}+3\cdot 4^n}{3^n+2^{2n+1}}$

(g) $a_n = \dfrac{6^n+2^n\cdot 3^{n-1}+5^n}{4^n-2\cdot 6^{n-1}}$

(h) $a_n = \dfrac{\left(\frac{1}{2}\right)^n-1}{\left(\frac{1}{3}\right)^{2n+1}-2}$

(i) $a_n = \sqrt[n]{5n^7-n^2+2}$

(j) $a_n = \sqrt[n]{2^n+3^{n-1}+2}$

(k) $a_n = 3n+\sqrt{n^2+4}$

(l) $a_n = n-\sqrt{n^2+4}$

(m) $a_n = n-\sqrt{2n^2-1}$

(n) $a_n = 2n-\sqrt{4n^2-n}$

(o) $a_n = n\left(n-\sqrt{n^2+2}\right)$

(p) $a_n = n\left(n-\sqrt{n^2+2n}\right)$

(q) $a_n = n-\sqrt[3]{n^3-n^2}$

(r) $a_n = n-\sqrt[3]{n^3-n^2}$

(s) $a_n = \dfrac{n-\sqrt{n^2+2n}}{n+1}$

(t) $a_n = \dfrac{n-\sqrt{n^2-1}}{2n-\sqrt{4n^2+n}}$

(u) $a_n = \dfrac{\sin n!}{n+1}$

(v) $a_n = \left(\dfrac{n+1}{n-1}\right)^{7n}$

(w) $a_n = \left(\dfrac{n^2+3}{n^2}\right)^{2n^2}$

(x) $a_n = \left(\dfrac{n+3}{4-n}\right)^n$

(y) $a_n = \left(1-\dfrac{1}{n^2}\right)^n$

(z) $a_n = \dfrac{3^{n+1}\cdot n!}{(n+1)!3^n}$

Odpowiedzi:

(a) 2
(b) $+\infty$
(c) 0
(d) $\frac{1}{2}$
(e) $\frac{1}{3}$
(f) $\frac{3}{2}$
(g) –4
(h) $\frac{1}{2}$
(i) 1
(j) 3
(k) $+\infty$
(l) 0
(m) $-\infty$
(n) $\frac{1}{4}$
(o) –1
(p) $-\infty$
(q) $-\infty$
(r) $\frac{1}{3}$
(s) 0
(t) 0
(u) 0
(v) e^{14}
(w) e^6
(x) nie istnieje
(y) 1
(z) 0

Zadanie 2

Obliczyć granicę ciągu (a_n), gdy:

(a) $a_n = \dfrac{n^2+1}{n}$

(b) $a_n = \left(\dfrac{n+1}{2n}\right)^n$

(c) $a_n = \dfrac{1+\frac{1}{2}+\frac{1}{4}+\ldots+\frac{1}{2^n}}{1+3+5+\ldots+(2n-1)}$

(d) $a_n = \left(\dfrac{n^2+1}{n}\right)^{\frac{n}{1-n}}$

(e) $a_n = 1+2^n-3^n$

(f) $a_n = \left(\dfrac{2n+1}{n}\right)^{n+1}$

(g) $a_n = \left(\dfrac{3n+1}{3n+2}\right)^{6n}$

(h) $a_n = \left(\dfrac{n}{n+1}\right)^n$

(i) $a_n = \left(1+\dfrac{1}{n}\right)^{3n-1}$

(j) $a_n = \left(\dfrac{n+4}{n+3}\right)^{5-2n}$

(k) $a_n = \left(\dfrac{n^2-1}{n^2}\right)^{2n^2-3}$

(l) $\lim\limits_{n\to\infty}\sqrt[n]{\dfrac{3^n+2^n}{5^n+4^n}}$

Odpowiedzi

(a) ∞
(b) 0
(c) 0
(d) 0
(e) $-\infty$
(f) ∞
(g) e^{-2}
(h) $\frac{1}{e}$
(i) e^{3}
(j) $\frac{1}{e^{2}}$
(k) e^{-2}
(l) $\frac{3}{5}$

Zadanie 3

Korzystając z twierdzenia o trzech ciągach oblicz granice:

(a) $\lim_{n\to\infty} \frac{2n+(-1)^n}{3n+2}$

(b) $\lim_{n\to\infty} \sqrt[n]{\frac{3^n+2^n}{5^n+4^n}}$

(c) $\lim_{n\to\infty} \frac{2n^2+\sin n!}{4n^2-3\cos n^2}$

(d) $\lim_{n\to\infty} \sqrt[n]{3+\sin n}$