

# Całki nieoznaczone

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## Zad.1 Obliczyć całki wymierne

$$\int \frac{dx}{(3x-2)^4} \quad I = \frac{-1}{9(3x-2)^3} + C$$

$$\int \frac{3x-4}{x^2-x-6} dx \quad I = \ln|x-3| + 2 \ln|x+2| + C$$

$$\int \frac{2x+6}{2x^2+3x+1} dx \quad I = 5 \ln \left| x + \frac{1}{2} \right| - 4 \ln|x+1| + C$$

$$\int \frac{11+5x}{x^2+3x-10} dx \quad I = 3 \ln|x-2| + 2 \ln|x+5| + C$$

$$\int \frac{dx}{6x^2-13x+6} \quad I = \ln \left| \frac{2x-3}{3x-2} \right| + C$$

$$\int \frac{x-1}{4x^2-4x+1} dx \quad I = \frac{1}{4} \left( \ln|2x-1| + \frac{1}{2x-1} \right) + C$$

$$\int \frac{dx}{2x^2-2x+5} \quad I = \frac{1}{3} \arctg \frac{2x-1}{3} + C$$

$$\int \frac{10x-44}{x^2-4x+20} dx \quad I = 5 \ln|x^2-4x+20| - 6 \arctg \frac{x-2}{4} + C$$

$$\int \frac{4x-5}{x^2-6x+10} dx \quad I = 2 \ln|x^2-6x+10| + 7 \arctg(x-3) + C$$

$$\int \frac{2x^2+7x+20}{x^2+6x+25} dx \quad I = 2x - \frac{5}{2} \ln|x^2+6x+25| - \frac{15}{4} \arctg \frac{x+3}{4} + C$$

$$\int \frac{x^3+2x-6}{x^2-x-2} dx \quad I = \frac{1}{2}x^2 + x + 3 \ln|x+1| + 2 \ln|x-2| + C$$

$$\int \frac{6x^3+4x+1}{x^4+x^2} dx \quad I = 4 \ln|x| + \ln|x^2+1| - \frac{1}{x} - \arctg x + C$$

## Zad.2 Obliczyć całki funkcji niewymiernych

$$\int \frac{x^2+1}{\sqrt{3x+1}} dx \quad I = \frac{2}{405} (27x^2 - 12x + 143) \sqrt{3x+1} + C$$

$$\int \frac{\sqrt{x}}{x-1} dx \quad I = 2\sqrt{x} + \ln \left| \frac{\sqrt{x}-1}{\sqrt{x}+1} \right| + C$$

$$\int \frac{1 + \sqrt{x}}{1 - \sqrt{x}} dx \quad I = -(x + 4\sqrt{x} + \ln|1 - \sqrt{x}|) + C$$

$$\int \frac{dx}{\sqrt{x} + 2\sqrt[3]{x^2}} \quad I = \frac{3}{2} \left( \sqrt[3]{x} - \sqrt[6]{x} + \frac{1}{2} \ln|2\sqrt[6]{x} + 1| \right) + C$$

Zad.3 Obliczyć całki funkcji zawierających  $\sqrt{ax^2 + bx + c}$

$$\int \frac{x}{\sqrt{1 - 2x - 3x^2}} dx \quad I = -\frac{1}{3}\sqrt{1 - 2x - 3x^2} - \frac{1}{9}\sqrt{3} \arcsin \frac{1}{2}(3x + 1) + C$$

$$\int \frac{2x - 3}{\sqrt{3 - 2x - x^2}} dx \quad I = 2\sqrt{3 - 2x - x^2} - 5 \arcsin \frac{1}{2}(x + 1) + C$$

$$\int \frac{3x + 2}{\sqrt{x^2 - 5x + 19}} dx \quad I = 3\sqrt{x^2 - 5x + 19} + \frac{19}{2} \ln \left| x - \frac{5}{2} + \sqrt{x^2 - 5x + 19} \right| + C$$

$$\int \frac{3x - 4}{\sqrt{4x^2 + 5x - 8}} dx \quad I = \frac{3}{4}\sqrt{4x^2 + 5x - 8} - \frac{47}{16} \ln \left| 2x + \frac{5}{4} + \sqrt{4x^2 + 5x - 8} \right| + C$$

$$\int \sqrt{x^2 - 3x + 2} dx \quad I = \frac{1}{2} \left( x - \frac{3}{2} \right) \sqrt{x^2 - 3x + 2} - \frac{1}{8} \ln \left| x - \frac{3}{2} + \sqrt{x^2 - 3x + 2} \right| + C$$

$$\int \frac{x^2}{\sqrt{1 - x^2}} dx \quad I = -\frac{1}{2} \times \sqrt{1 - x^2} + \frac{1}{2} \arcsin x + C$$

Zad.4 Obliczyć całki funkcji trygonometrycznych i cyklometrycznych

$$\int \cos x \cos 3x dx \quad I = \frac{1}{8} \sin 4x + \frac{1}{4} \sin 2x + C$$

$$\int \frac{\sin x dx}{\sqrt[3]{1 + 2 \cos x}} \quad I = -\frac{3}{4} \sqrt[3]{(1 + 2 \cos x)^2} + C$$

$$\int \frac{dx}{\sin^3 x \cos x} \quad I = -\frac{1}{2 \sin^2 x} + \ln |\operatorname{tg} x| + C$$

$$\int \frac{dx}{\sin x} \quad I = \ln \left| \operatorname{tg} \frac{1}{2} x \right| + C$$

$$\int \frac{dx}{\sin^2 x \cos^4 x} \quad I = \frac{1}{3 \sin x \cos^3 x} - \frac{8}{3} \operatorname{ctg} 2x + C$$

$$\int \frac{\arcsin x}{\sqrt{(1 - x^2)^3}} dx \quad I = \frac{x \arcsin x}{\sqrt{1 - x^2}} + \frac{1}{2} \ln |1 - x^2| + C$$

$$\int \frac{x^2}{1 + x^2} \operatorname{arctg} x dx \quad I = x \operatorname{arctg} x - \frac{1}{2} \ln |1 - x^2| - \frac{1}{2} (\operatorname{arctg} x)^2 + C$$