

Podstawowe wzory na całki

$$1. \int c \, dx = cx + C, \quad C \in \mathbb{R}.$$

$$2. \int 0 \, dx = 0;$$

$$3. \int x^\alpha \, dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \quad \text{dla } \alpha \neq -1;$$

$$4. \int \frac{1}{x} \, dx = \ln |x| + C, \quad \text{dla } x \neq 0.$$

$$5. \int \sin x \, dx = -\cos x + C;$$

$$6. \int \cos x \, dx = \sin x + C;$$

$$7. \int \frac{1}{\sin^2 x} \, dx = -\operatorname{ctg} x + C;$$

$$8. \int \frac{1}{\cos^2 x} \, dx = \operatorname{tg} x + C$$

$$9. \int a^x \, dx = \frac{a^x}{\ln a} + C$$

$$10. \int \frac{1}{1+x^2} \, dx = \operatorname{arctg} x + C;$$

$$11. \int \frac{1}{\sqrt{1-x^2}} \, dx = \arcsin x + C;$$

$$12. \int \frac{1}{\sqrt{a^2-x^2}} \, dx = \arcsin \frac{x}{a} + C;$$

$$13. \int \frac{1}{\sqrt{x^2+a^2}} \, dx = \ln |x + \sqrt{x^2+a^2}| + C;$$

$$14. \int \frac{1}{x^2-a^2} \, dx = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C;$$

$$15. \int \frac{1}{x^2+a^2} \, dx = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C;$$

Podstawienie uniwersalne:

$$t = \operatorname{tg} \frac{x}{2},$$

$$dx = \frac{2dt}{1+t^2},$$

$$\sin x = \frac{2t}{1+t^2}, \quad \cos x = \frac{1-t^2}{1+t^2}.$$

Podstawienie tangensowe:

$$t = \operatorname{tg} x,$$

$$dx = \frac{dt}{1+t^2},$$

$$\sin^2 x = \frac{t^2}{1+t^2}, \quad \cos^2 x = \frac{1}{1+t^2}.$$